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REPORT

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TRAJECTORY CHARACTERISTICS
DURING THE LEM MISSION (U)

CODE 26512

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(NASA-CR-117282) TRAJECTORY CHARACTERISTICS
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GLOSSARY OF SYMBOLS

<u>Symbol</u>	<u>Definition</u>	<u>Unit</u>
g	Thrust Acceleration/32.17	unitless
h	Altitude	ft. or n. mi.
r	Radial Position	ft. or n. mi.
t	Time	sec. or minutes
x,y,z	Position in Rectilinear Coordinates	ft.
$\dot{x}, \dot{y}, \dot{z}$	Velocity in Rectilinear Coordinates	ft/sec
γ	Flight Path Angle	degrees
θ	Pitch Angle	degrees
ϕ	Roll Angle	degrees
ψ	Yaw Angle	degrees
λ	Visibility margin measured from the window lower or upper edge to the LOS to the Landing-Site or horizon	degrees
μ	Gravity constant of the moon	ft ³ /sec ²
ζ	The angle between the z-Body Axis and the LOS to the horizon or to the landing-site	degrees
ρ	Slant range between LEM and the CSM	ft. or n. mi.
$\dot{\rho}$	Slant range-rate between LEM and the CSM	fps.
σ	Central angle measured in the plane of the trajectory	degrees
$\dot{\sigma}$	Rate of change of central angle measured in the plane of the trajectory	deg/sec
ω	Line-of-sight angle measured in the xz-body axis plane from the z-body axis to the projection of the LOS on the xz-body axis plane, Rendezvous Radar Shaft Angle	degrees
τ	Line-of-sight angle measured from the xz-body axis plane to the LOS, Rendezvous Radar Trunion Angle	degrees
E	Elevation angle measured between the stable member z-Axis and the LOS to the CSM	degrees
R	Surface Range	ft. or n. mi.

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V. LEM Normal Error Performance

Normal error performance is the likely trajectory deviations from the nominal trajectory caused by in-tolerance errors in the following categories;

1. Initiation of the navigation system - Prior to separation of the LEM from the CSM, the command module navigation data is transferred to the LEM. The transferred information on the velocity and position is used as initial conditions to the LEM integrating inertial system. This data is only as accurate as the estimated CSM orbit via the orbital navigation technique.
2. Navigation system sensor errors - This group of error sources includes such uncertainties as the alignment of the IMU, platform drift rates and radar accuracies.
3. Errors in execution - This group of errors includes the uncertainties in the thrust and attitude from the levels commanded by the guidance system.

The combined effects of all the errors sources presented under the three categories briefly identified above are analyzed using a linear statistical error technique developed at GAEC. References V-1 thru V-8 discuss in detail the mathematical theory used as a basis to generate the data presented in this section. At this time, only the results of an analysis performed on the powered descent trajectory are presented.* Future trajectory reports will include the results of studies on all phases of the LEM mission.

A. Error Sources

Table V-1 presents the uncertainties in the LEM inertial system's indication of the position and velocity at pericyynthion. These errors are presented

TABLE V-1

COVARIANCE MATRIX OF UNCERTAINTIES AT PERICYNTHION

x	y	z	\dot{x}	\dot{y}	\dot{z}	
4.6519 E 05	1.7127 E 04	1.4045 E 06	-1.3300 E 03	8.0413 E 00	4.9294 E 01	x
1.7127 E 04	3.9102 E 05	3.2212 E 03	-9.4216 E 00	4.5204 E 01	-1.4251 E 01	y
1.4045 E 06	3.2212 E 03	7.9014 E 06	-7.5680 E 03	1.4930 E 02	1.4911 E 03	z
-1.3300 E 03	-9.4216 E 00	-7.5680 E 03	7.4018 E 00	-1.2291 E -01	-1.4635 E 00	\dot{x}
8.0413 E 00	4.5204 E 01	1.4920 E 02	-1.2291 E -01	3.5357 E -01	3.9890 E -02	\dot{y}
4.9294 E 01	-1.4251 E 01	1.4911 E 03	-1.4635 E 00	3.9890 E -02	5.0063 E -01	\dot{z}

in the form of a covariance matrix in which the diagonal terms are the variances in the components of position and velocity, and the off-diagonal terms are the covariances. The off-diagonal terms indicate the correlation between the errors on the diagonal. The matrix is referenced to the stable member axis system; that is, starting at the upper left hand element of the matrix, and in successive order, the variances in the position along the x, y and z axis, and in the velocity along the x, y

* Errors caused by IMU sensor errors, timing errors and control execution errors during descent insertion will cause increases in errors along the powered descent from those presented in this report.

characteristics of the last 10 seconds of the trajectory are not available. However, the total deviation in fuel consumption required for correction of guidance errors should be approximately equal to the last value recorded on figure V.C-7. The additional fuel required for correction of guidance errors, assuming normal error performance, is 30 lbs. (3 sigma).

Figure V.C-8 thru V.C-15 present the weighting factors used during the two update periods. ($270 \leq t \leq 290$, $3350 \leq t \leq 430$) These weighting factors were generated using an optimum technique for minimizing the variance in a linear combination of inertial and radar information. (see Reference V.C-8). This technique is sometimes referred to as "Linear Filtering" or "Kalman Filtering."

VI. Summary of Normal Error Performance

Only errors accrued along the powered descent trajectory have been presented in this report. The 1σ errors in the LEM position, velocity and fuel consumption at the hover point are:

Altitude = 25 ft (1σ)
Downrange = 2850 ft (1σ)
Cross track = 785 ft (1σ)
Vertical velocity = 1.5 ft/sec (1σ)
Horizontal velocity = 1.8 ft/sec (1σ)
Cross track velocity = 1.5 ft/sec (1σ)
Fuel consumption = 10 lbs (1σ)

Future errors studies on the powered descent trajectory will include the primary navigation and guidance update procedure and the errors caused by sensor errors during the insertion. Other phases of the LEM mission will also be treated.

C. Error Results

When discussing the errors in position and velocity, a subscript "I" denotes an uncertainty in the navigation system and a variable without a subscript denotes a trajectory deviation from the nominal. All errors in state (e.g. velocity and position) are referenced to the stable member axis system.

Figure V.C-1 is a time history of the one sigma uncertainty in navigation information and of one sigma trajectory deviations in the x-axis direction. As the landing-site is approached, the x-axis errors closely approximate errors in altitude. For the first 270 seconds into the powered descent, the trajectory deviations are approximately equal to the uncertainties in the inertial system. This should be expected since the guidance law is using information from the navigation system to generate the vehicle control commands. At 270 seconds (or approximately 25,000 ft altitude), an update is made by mixing landing-radar and inertial information. An improvement in the navigation accuracy is immediately introduced. However, the trajectory deviation slowly reduces to the accuracy of the navigation system at the end of the Fuel Optimum Phase. At the beginning of the Visibility Phase, an improvement in the indicated state is again made resulting in an altitude uncertainty and trajectory deviation at the hover of approximately 25 ft (1σ). The final error in altitude can be improved if the updating procedure is continued further into the trajectory than was done for this analysis.

Figure V.C-2 presents the navigation uncertainties and trajectory deviations in velocity along the stable member x-axis. After the first update period, an increase in velocity deviation of 14.5 ft/sec is expected due to the guidance response to a detected error in altitude. A less severe velocity response can be expected during the second update phase.

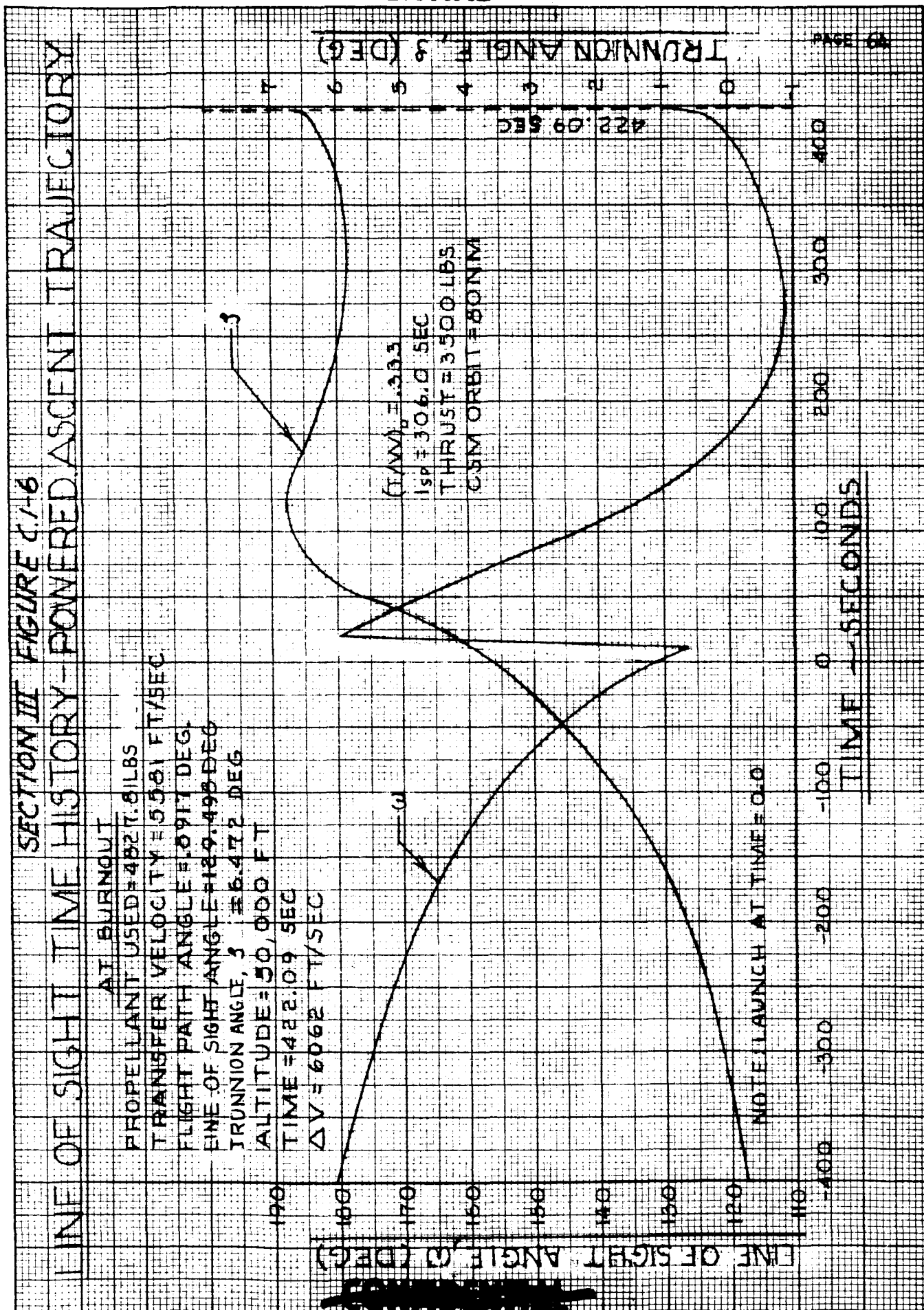
Figure V.C-3 presents the errors in the y and z direction. The y-axis corresponds to cross track at the landing-site, and the z-axis corresponds to down range. No improvement in down range or cross track accuracy can be expected due to an update. However, the rate of increase in the likely position errors is stopped. The update in velocity along the y and z axes prevents any further increase in position uncertainties.

Figure V.C-4 shows the navigation uncertainties and trajectory deviations in the velocities along the y and z axes. The increase in z-axis velocity trajectory deviations is due to the thrust uncertainties.

Figure V.C-5 is a time history of the expected deviations in the thrust command signal from the nominal. At each discontinuity point in navigation information, a corresponding discontinuity in the thrust command is likely. The maximum of 1σ thrust deviations is 260 lbs and can be expected to occur prior to initiation of VP. Figure V.C-6 shows the expected deviation in attitude command signals. The maximum of 1σ attitude deviations occurs at 270 seconds into the powered descent trajectory, and has a 1σ value at that time of 2.9 degrees.

Figure V.C-7 presents the expected deviation in fuel consumption along the descent trajectory. At this time, methods of analyzing the error

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and z axes of the stable member coordinate system are defined. This covariance matrix was obtained by propagating the covariance matrix, representing the orbital navigation uncertainties, from insertion to the pericynthion. The orbital navigation uncertainties at insertion were obtained through correspondence with MIT/IL (Reference V-9). At insertion, the uncertainties were given as:

Altitude	770 ft	(1 sigma)
Down Range	2380 ft	(1 sigma)
Cross Track	673 ft	(1 sigma)
Vertical Velocity	1.71 ft/sec	(1 sigma)
Horizontal Velocity	.56 ft/sec	(1 sigma)
Cross Track Velocity	.552 ft/sec	(1 sigma)

for the MIT/IL "Model-2" orbital navigation technique. (Reference V-10).

The variances in the trajectory state deviations from the nominal at powered descent initiation are assumed to be 10% larger in magnitude than the variances of errors in the navigation system, and are assumed to have approximately 90% correlation with the navigation uncertainties. The high correlation asserts that trajectory deviations at pericynthion of the descent orbit are primarily due to the uncertainties in navigation. The magnitude of the trajectory deviations are assumed to be slightly larger to account for possible errors in the execution of the insertion maneuver. A detailed account of the meaning of the correlation of the navigation uncertainties with trajectory deviations can be found in reference V-7.

Table V-2 presents the IMU sensor errors considered in the analysis. The one sigma values quoted in Table V-2 were taken from an MIT/IL documentation of the inertial system performance presented in reference V-11. The methods by which sensor errors are incorporated into a linear statistical error analysis can be found in reference V-3.

TABLE V-2

IMU SENSOR ERRORS (1-Sigma Values)

Accelerometer zero bias	= .2 cm/sec ²
Accelerometer scale factor error	= 100 ppm
Accelerometer sensitivity (1 st order)	
Scale factor error	= 10 ppm/g
Initial stable member	
Misalignment	= .82 mr
Fixed Drift Rate	= .15 deg/hr.

Table V-3 presents the errors in execution assumed in the analysis. A 0.1 degree (3σ) uncertainty in attitude control is attributed to the RCS dead band. A 100 lbs. (3σ) thrust uncertainty is attributed to engine throttle resolution.

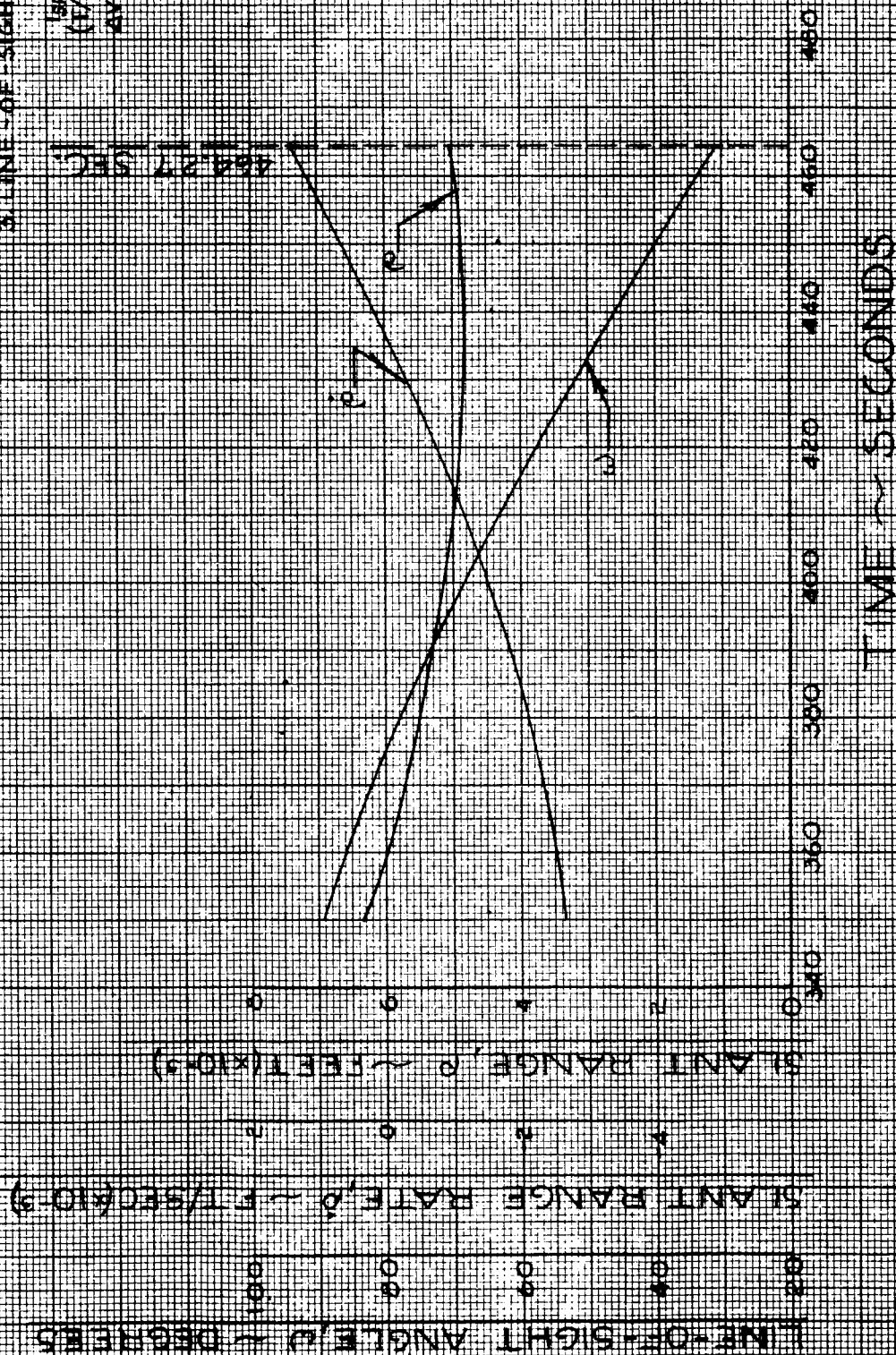
SECTION III FIGURE B.7.2-6

VISIBILITY PHASE

POWERED DESCENT

TRAJECTORY PARAMETERS
1. SLANT RANGE
2. SLANT RANGE RATE
3. LINE-OF-SIGHT ANGLE

187.301 SEC
(TAN) = 0.274
AV = 1088 FT/SEC



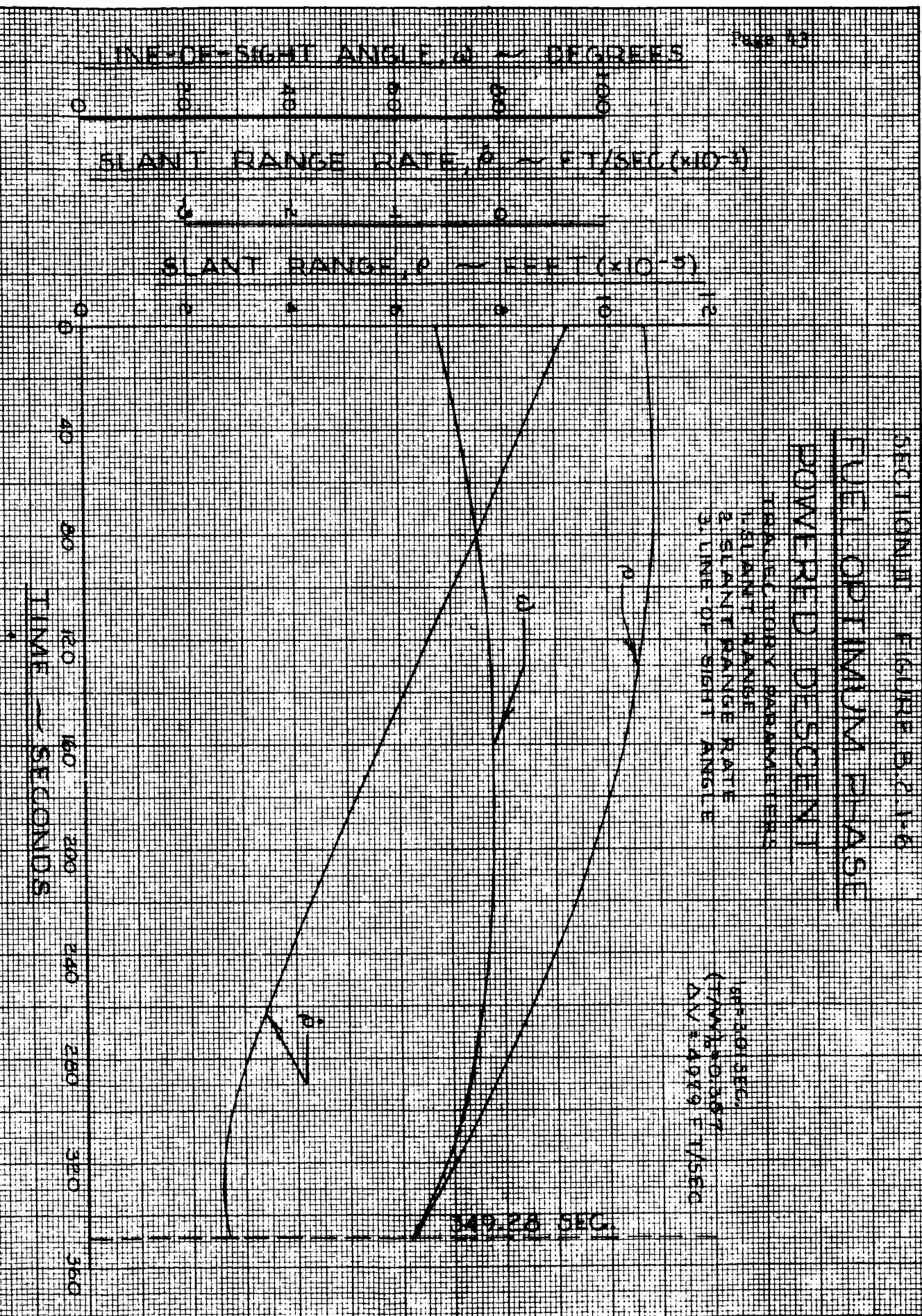


TABLE V-3

 ERROR IN EXECUTION

 (3-sigma)

Attitude Errors	= .1 deg.
Thrust Errors	= 100 lbs

The errors quoted in the PNGS requirements (Reference V-12) for the landing radar were used in the analysis of the latter portions of the powered descent trajectory. The radar error in the measurement of altitude is assumed to be one percent of the vehicle altitude or 5 ft, which ever is greater (3σ). The velocity errors are assumed to one percent of the vehicle velocity or 1 ft/sec, which ever is greater (3σ). The uncertainties for the radar are calculated using the assumption of a smooth lunar surface with a radius equal to the mean. The effects of surface variations will be studied in the future.

B. N & G Operational Procedures

For powered descent, the navigation and guidance procedures assumed in the error analysis are in chronological order:

1. Alignment of the IMU using the AOT is made fifteen minutes prior to the initiation of powered descent. The uncertainty in the alignment at the initiation of powered descent is 0.82 milliradians. (1σ).
2. The inertial navigation system is used to supply the descent explicit guidance equations with information required to generate control signals to the RCS.
3. At 270 seconds into the powered descent trajectory (approximately 25,000 ft altitude), an update of the inertial system is made using information on altitude and velocity from the landing radar.
4. Radar - Inertial Mixing is terminated at $t = 290$ seconds into the powered descent.
5. From 290 seconds to the end of FOP, inertial information is used exclusively.
6. From the beginning of the VP to 430 seconds, mixing of radar - and inertial information is again made.
7. From $t = 430$ seconds to the end of VP, inertial information is used exclusively.